Parameter Estimation of a Rotary Inverted Pendulum

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IMPORTANT NOTICES

Acknowledgements

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ABSTRACT

A step up and step down test is employed to estimate the parameters of a rotary inverted pendulum. The data collected are used to fit a nonlinear model of the inverted pendulum. Two different methods of estimating the parameters of the pendulum are investigated. In addition, the effects of varying the amplitude of the test signal are investigated.

INTRODUCTION

The inverted pendulum system is widely used in control study. A rotary arm (Fig 1) version of the inverted pendulum system is used. The apparatus (Kri PP-300) is supplied by Kent Ridge Instruments [1].

There are two important steps towards designing a control system: plant modeling and controller design. In the case of the rotary inverted pendulum, the plant model is described by two non-linear equations. To design a controller which could balance the pendulum in the upright position, a linearised model of the inverted pendulum is derived from the non-linear model. (See equation(2).)

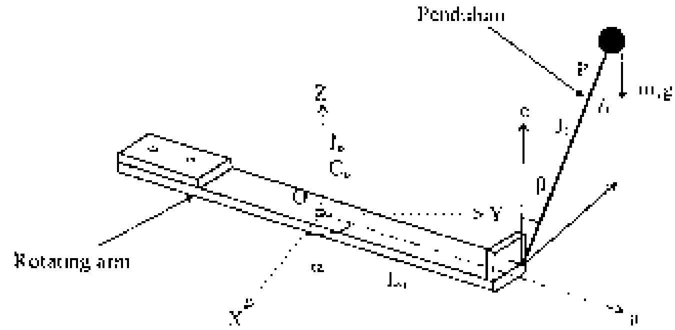

Fig 1 The Rotating Arm and the Pendulum

THEORY

There are two methods for estimating the parameters of the model of the rotary inverted pendulum described by equation (1) [2,3]. The first method does not require the user to know the actual physical parameters: m_1, l_1 and L_0. The parameters a, b, d, g, m and n are estimated using the least square method. The second method needs physical parameters such as L_0, l_1 and m_1 to be measured. A set of values for J_0, C_0, J_1 and C_1 can be determined.

$$\begin{bmatrix} J_0 + m_1L_0^2 + m_1l_1^2 \sin^2\beta & m_1l_1L_0\cos\beta \\ m_1l_1L_0\cos\beta & J_1+m_1l_1^2\end{bmatrix}\begin{bmatrix}\ddot\alpha\\ \ddot\beta\end{bmatrix} + \begin{bmatrix}0\\ -m_1l_1g_1\sin\beta\end{bmatrix}$$

$$\begin{bmatrix} C_0 + \frac{1}{2} m_1l_1^2\sin\dot\beta(2\beta) & -m_1l_1L_0\sin\beta + \frac{1}{2} m_1l_1^2\sin\dot\alpha(2\beta) \\ -\frac{1}{2} m_1l_1^2\sin\dot\alpha(2\beta) & C_1\end{bmatrix}\begin{bmatrix}\dot\alpha\\ \dot\beta\end{bmatrix} = \begin{bmatrix}\tau\\ 0\end{bmatrix} \quad \textbf{(1)}$$

where,
J_0-moment of inertia of rotating arm C_0-friction coefficient of rotating arm
J_1-moment of inertia of pendulum C_1-friction coefficient of pendulum
α - arm velocity α – arm velocity α - arm acceleration
β - pendulum position β – pendulum velocity β - pendulum acceleration
m_1-mass of pendulum l_1-length of pendulum K_t-torque constant
L_0-length of rotating arm g_1-gravitional force K_b-back emf constant
Ra-terminal resistance

Equation can be linearised and written in the state space form as shown below[2]:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$
$$y = C\mathbf{x}$$

$$A = \begin{bmatrix} \frac{d(C_0+n)}{b^2-ad} & \frac{bg}{b^2-ad} & \frac{-bC_1}{b^2-ad} \\ 0 & 0 & 1 \\ \frac{-b(C_0+n)}{b^2-ad} & \frac{-ag}{b^2-ad} & \frac{aC_1}{b^2-ad} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{-dm}{b^2-ad} \\ 0 \\ \frac{bm}{b^2-ad} \end{bmatrix}, \quad x = \begin{bmatrix}\dot\alpha \\ \beta \\ \dot\beta\end{bmatrix} \qquad \textbf{(2)}$$

and $a = J_0+m_1L_0^2$ $b = m_1l_1L_1$ $d = J_1+ m_1l_1^2$ $g = m_1l_1g_1$

$$m = \frac{K_tK_a}{R_a} \qquad n = \frac{K_tK_b}{R_a}$$

u is the PWM input signal and C is the matrix that reflects the available measurements from the inverted pendulum apparatus. In this case,

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

From equation(2), the correct signs for A and B matrices are as follows :

$$A = \begin{bmatrix} - & - & + \\ 0 & 0 & 1 \\ + & + & - \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} + \\ 0 \\ - \end{bmatrix}$$

RESULTS AND OBSERVATION

I) At test amplitude of 60 (the nominal level), the experiment was conducted ten times. Three sample sets of data are shown:

Method 1	Data1_60	Data2_60	Data3_60

A matrix	-0.9087 -0.8424 0.004 0 0 1 -2.1418 46.3336 -0.2184	-0.8224 -1.0230 0.0053 0 0 1 -3.5929 43.6578 -0.2247	-0.7447 -0.9892 0.0041 0 0 1 -3.8688 43.4358 -0.1798
B matrix	0.3776 0 0.8901	0.3501 0 1.5295	0.3376 0 1.7538
Gain, K	-14.809, 945.518, 66.047	-15.763, 552.319, 38.7052	-15.979, 474.990, 33.304
Open loop poles	-6.8909 -0.9483 6.7056	-6.6728 -0.9082 6.5269	-6.6302 -0.8343 6.5399

Method 2			
A matrix	-0.4218 -0.3964 0.0027 0 0 1 0.2841 48.3204 -0.3264	-0.4333 -0.2696 0.0019 0 0 1 0.2917 48.2116 -0.3428	-0.4523 0.2607 -0.0017 0 0 1 0.3081 48.4230 -0.3234
B matrix	0.0603 0 -0.0406	0.0411 0 -0.0276	-0.0392 0 0.0267
*Gain, K	-0.0849, -20.648,-1.4361	-0.1252, -30.3528, -2.1106	0.1299, 31.369, 2.1794
Open loop poles	-0.4195 6.7889 -7.1177	-0.4317 6.7734 -7.1179	-0.4539 -7.1214 6.7996

*-The gain is scaled to a factor of 0.001

Method 1: The a_{31} element in A matrix and the last element in the B matrix have the wrong signs for the ten experiments. This method gives reasonable controller gain except for the first set of data because the gain is exceptionally large. There might be experimental error when the experiment is being conducted. The method also gives the desired open-loop poles: a pair of conjugate poles and the third pole on the left-half side of the plane. Fig. 2 shows that the open loop poles obtained from the ten experiments are clustered at the same regions.

Method 2: Some of the elements of the of A and B matrices have inconsistent signs for the ten experiments. Although this method uses J_0, C_0, J_1 and C_1 in the calculations, the inconsistent elements are those using only J_0 (moment of inertia of rotating arm). Therefore the estimated J_0 could be wrong due to calculation errors. There is no convergence in the controller gain values. However the positions of the open-loop poles still correspond.

II) The experiment is conducted for different amplitudes of test signal at 30, 40, 60 and 85. The four conditions are used to analyze the effects of varying test amplitude on the two methods.

Method 1	Amplitude 30	Amplitude 40	Amplitude 85
A matrix	0.2581 -1.6290 -0.0260 0 0 1 -0.5525 37.6765 0.6011	-1.554 0.3996 -0.0015 0 0 1 -9.2128 52.248 0.1988	-0.9434 -1.0152 0.0066 0 0 1 1.5205 50.6282 -0.3287
B matrix	-0.0116 0 0.0248	0.3376 0 1.7538	0.4863 0 -0.7837
Open loop pole	-5.8340, 0.2342, 6.4590	-7.3732,-1.4808, 7.1009	-7.2995, -0.9124, 6.9397

Method 2	Amplitude 30	Amplitude 40	Amplitude 85
A matrix	0.0068 -0.9706 -0.0055 0 0 1 -0.0028 30.3947 0.1710	-0.3812 -0.1009 0.0005 0 0 1 0.2643 49.5280 -0.2253	-0.4904 -0.5340 0.0034 0 0 1 0.3372 49.4181 -0.3125
B matrix	0.2368 0 -0.0995	0.0149 0 -0.0103	0.0796 0 -0.0548
Open loop pole	0.0067 -5.4282 5.5994	-0.3807 6.9256 -7.1514	-0.4867 6.8736 -7.1898

For method 1, the results show that at test amplitude of 85, elements of A and B matrices have the correct signs. For all the test amplitudes, the controller gains and the open-loop poles are consistent. This method gives the correct controller design when the closed loop poles are varied.
For method 2, the matrices are not consistent. However the positions of the open-loop poles still correspond.
At test amplitude of 30 (barely able to rotate the arm), the model could not be estimated. As for amplitude 40, the model cannot be estimated accurately because of friction at the arm and pendulum.

CONCLUSION

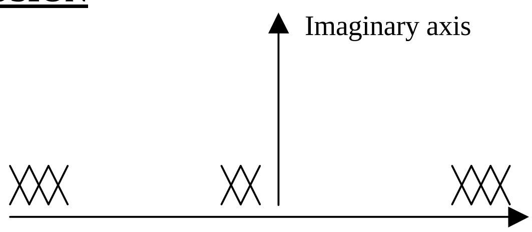

Fig. 2: Clusters of open loop poles

The two methods were tested extensively with more than twenty sets of data collected. Overall, both methods gave reasonable and correct open-loop poles. On the other hand, method 1 gives a better controller because the controller gain and open-loop poles are correct. Also, A and B matrices for method 1 is more consistent compared to method 2.

RECOMMENDATION

The wrong signs in A and B matrices in method 1 provide an area for further research. The experiments should be conducted as many times as possible (for a particular test amplitude) because the arm starts rotation at different points. This might affect the values of the estimated parameters for both methods 1 and 2. Special attention may be given to test amplitudes when the arm starts to rotate and when the arm starts to swing violently.

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